Comments on "Limiting Velocity for a Rotating Plasma"[1]

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In considering the rotating plasma experiments of Fahleson[2], we have constructed a boundary-layer type of analysis[3] which is believed to be applicable to such experiments for sufficiently high initial pressure (greater than about 50 μ Hg for Fahleson's apparatus). The general agreement between our theory and Fahleson's experiment is fair, but for low currents the theory predicts that the azimuthal velocity is only slightly more than half the ionization speed, $(2w_i/m)^{1/2}$, where w_i is the ionization energy and m the atomic mass. In contrast, the low-density theory of Lin[1] predicts a velocity somewhat in excess of the ionization speed, depending upon the magnitude of the radiation loss, which is in much closer agreement with the experiments. While it is to be expected that different results will be obtained from a high pressure, continuum theory and a low-pressure, rarefied flow theory, it is our opinion that the continuum theory is more applicable to Fahleson's experimental conditions for reasons which are too extensive to be discussed here. Upon reexamining Lin's theory, we have found that inadequate consideration has been given to the effect of charge-exchange collisions. We show below what is considered to be a proper accounting for this effect, and note that at low currents the azimuthal velocity predicted by this corrected treatment is less than the ionization speed and even less than that predicted by the continuum theory.

Our initial physical assumptions are identical to those of Lin. We consider a plane parallel flow of plasma between two plane parallel electrodes spaced apart a distance d. There exists a uniform magnetic field, of flux density B, having a direction normal to the plane of the flow and parallel to the electrodes. All mean free paths are large compared with the spacing d. It is further assumed that there is a stationary background of cold, neutral particles, which are in good thermal contact with the cold, stationary electrodes. When a current I flows between the electrodes, the plasma component (ion-electron pairs) is accelerated to a mean velocity v, the neutrals remaining stationary. Simultaneously, the resistive heating of the plasma raises the electron temperature, thereby ionizing some of the neutrals by electron impact. The newly formed, stationary ions must then be accelerated to the velocity v. Also, energy and momentum are lost from the plasma by collisions with the neutrals and by a flow of charged particle pairs to the electrodes, where they recombine to form atoms.

Our assumption concerning the loss of energy to the wall as a result of charge-exchange collisions differs from that of Lin. We propose that such collisions produce neutral atoms of speed v and temperature equal to the ion temperature T_i, which without further collision impinge upon the wall and deposit an energy $mv^2/2 + 2kT_i$ for each charge-exchange collision. (We use here the free molecule flow value of 2kT for the thermal energy per particle impinging upon a wall.) At any instant, the number of such high speed neutral atoms born in charge-exchange collisions, but not yet having reached the wall, must be much less than the number of cold, stationary neutral atoms, because the mean speeds of the two classes of neutrals are so different even if their flow rates to the wall are comparable. We therefore compute separately the energy and momentum loss rates caused by these high speed neutrals and do not consider them a part of the ion loss rates, as did Lin.

We begin by considering a fixed volume of fluid containing N_i singly-charged ions (and hence N_i electrons) and N_a atoms. If Γ_i is the rate of production of ions and Γ_c the rate of charge-exchange collisions in this volume, while Λ_i is the number flux of ion-electron pairs and Λ_c the number flux of high speed atoms (born in charge exchange collisions) impinging on the walls surrounding this volume, then the conservation of ionic species and high-speed atoms requires that:

$$\frac{dN_i}{dt} = \Gamma_i - \Lambda_i \tag{1}$$

$$\Lambda_c = \Gamma_c \tag{2}$$

The conservation of momentum becomes:

$$\frac{d}{dt}(mvN_i) = IBd - mv(\Lambda_i + \Lambda_c) \tag{3}$$

which has the same form as Lin's Eq. (4). However, for energy conservation for the whole fluid we have:

$$\frac{d}{dt}\left(\{\frac{1}{2}mv^2 + w_i + \frac{3}{2}k(T_i + T_e)\} N_i\right) = VI - P$$

$$- \{\frac{1}{2}mv^2 + w_i + 2k(T_i + T_e)\} \Lambda_i - \{\frac{1}{2}mv^2 + 2kT_i\} \Lambda_c \qquad (4)$$

in which V is the electrode potential difference and P is the radiation power loss.

There is no contribution from the slow neutral particles to either the momentum or energy equation since they have negligible energy or momentum. However, the high speed neutral particles contribute an energy and momentum loss to the wall but negligible energy or momentum content to the volume, since they are so few in number.

Following Lin, we neglect the time derivative of ln $(T_e + T_i)$ compared with that of ln N_i, and assume that V = IBd, i.e., that the resistive loss in the plasma is small compared with IBd. As a consequence, Eqs. (1) - (4) may be combined to give:

$$\frac{1}{2}mv^2 = \{w_i + \frac{3}{2}k(T_e + T_i) + P/\Gamma_i\} \Gamma_i/(\Gamma_i + \Gamma_c)$$

$$+ \frac{1}{2}k(T_e + T_i) \Lambda_i/(\Gamma_i + \Gamma_c) + 2kT_i\Gamma_c/(\Gamma_i + \Gamma_c) \qquad (5)$$

which differs from Lin's Eq. (8) by the factor of $\Gamma_i/(\Gamma_i + \Gamma_c)$ appearing in the first term on the right hand side[4] and the additional terms on the right.

It will now be shown that the charge-exchange collision rate Γ_c greatly exceeds the electron impact ionization rate Γ_i unless the electron temperature is as great as w_i/k, with the result that Eq. (5) predicts a flow velocity v which depends upon T_i and not upon w_i. This disparity in Γ_i and Γ_c may be seen by considering the following estimates of these rates:

$$\Gamma_i = N_a N_i Q_i \bar{v}_e \, e^{-w_i/kT_e} \tag{6}$$

$$\Gamma_c = N_a N_i Q_c \bar{v}_i \tag{7}$$

in which Q_i is the ionization cross section, Q_c the charge-exchange cross-section, and $\bar{v}_e$ and $\bar{v}_i$ the mean speeds of the electrons and ions, respectively. Now $Q_c \simeq 50 Q_i$ while $\bar{v}_e \simeq 100 \bar{v}_i$. Thus the pre-exponential factor of Eq. (6) about equals the right hand side of Eq. (7), with the result that

$$\Gamma_i \simeq \Gamma_c \, e^{-w_i/kT_e} \tag{8}$$

For $kT_e \ll w_i$ and $T_i \simeq T_e$, Eq. (5) and (8) combine to give, for the steady state,

$$\frac{1}{2} mv^2 \simeq 2kT_i \tag{9}$$

The electron and ion temperatures are not determined by the foregoing equations, but must be found from a detailed energy balance for the electrons. The electron temperatures so calculated by Lin for low or moderate degrees of ionization generally are small compared with w_i/k, and are not strongly dependent upon the degree of ionization (at least for low or moderate amounts of ionization). Ion temperatures calculated by Lin are in the range of 0.1 to 0.2 times w_i/k, so that Eq. (9) would predict velocities of 50% to 70% of the ionization speed, more or less independent of degree of ionization and hence current.

The continuum, or boundary-layer type of analysis[3], predicts a result similar to that of Eq. (9), under similar restrictions of moderate degree of ionization:

$$\frac{1}{2} mv^2 = \frac{5}{2} kT_i/Pr \tag{10}$$

in which Pr is the Prandtl number (2/3 for a monatomic gas) and $5/2\ kT_i$ is the enthalpy per ion, the continuum equivalent of $2kT_i$ for the free-molecular energy flux to the wall, used in Eq. (4). The difference between the right hand sides of Eqs. (9) and (10) are typical of those found in comparing continuum and rarefied flow drag or heat transfer.

If one accepts the above modification of the theory of Lin, then either the continuum or rarefied flow theory predicts azimuthal velocities less than those measured by Fahleson. It is our opinion that this discrepancy is most likely due to other effects present in the experiment which are not taken into account by either of the theories which assume a very simple flow pattern.

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Footnotes

1. S.-C. Lin, Phys. Fl. 4, 1277 (1961)
2. U. V. Fahleson, Phys. Fl. 4, 123 (1961)
3. P. Sockol, Sc. D. Thesis, Massachusetts Institute of Technology, Cambridge, Mass. (1967)
4. This difference would disappear if the coefficient of Λ_c in Eq. (4) were augmented (incorrectly) by $w_i + 3kT_e/2 - kT_i/2$.